ÉRETTSÉGI VIZSGA • 2005. május 10.

MATEMATIKA ANGOL NYELVEN MATHEMATICS

EMELT SZINTŰ ÉRETTSÉGI VIZSGA HIGHER LEVEL FINAL EXAMINATION

Az írásbeli vizsga időtartama: 240 perc
Time allowed for the examination: 240 minutes

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ MARKSCHEME

OKTATÁSI MINISZTÉRIUM
MINISTRY OF EDUCATION

Instructions to examiners

Formal requirements:

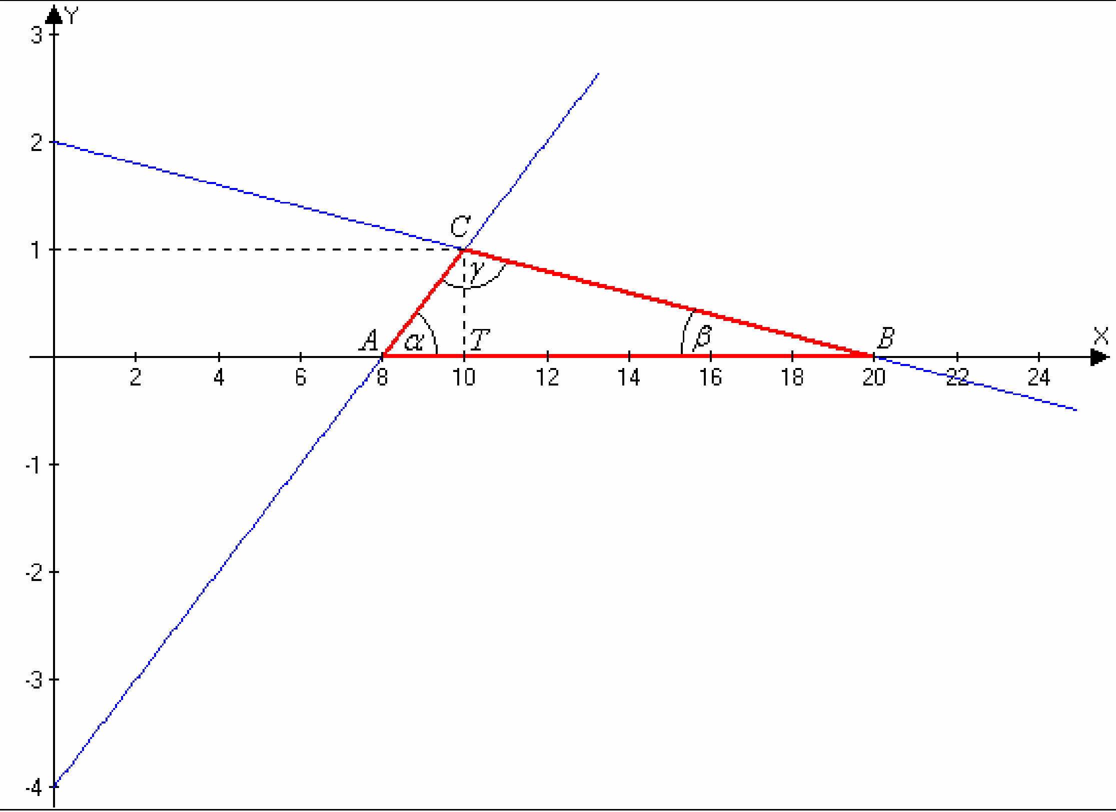
- Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc in the conventional way.
- The first one of the rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
- **If the solution is perfect,** it is enough to enter the maximum scores in the appropriate rectangles.
- If the solution is incomplete or incorrect, please indicate the individual **subtotals** on the paper, too.

Assessment of content:

- The markscheme contains more than one solution for some of the problems. If the solution by the candidate is different, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
- The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
- If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one in the markscheme.
- If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error is made. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
- **In the case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or in the next part of the problem and used correctly, the maximum score is due for the next part.
- Where the markscheme shows a **unit** in brackets, the solution should be considered complete without that unit as well.
- If there is more than one attempt to solve a problem, **assess only one** of them (the one that is worth the largest number of points).
- **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
- **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
- **Assess only four out of the five problems in Section II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1.



a)

The $y=0$ line, that is the x-axis, intersects the $x+10y=20$ line at point $B(20;\ 0)$, and	2 points	
the line $y=\frac{1}{2}x-4$ at point $A(8;\ 0)$.	2 points	
The solution of the simultaneous linear equations $x+10y=20$ and $y=\frac{1}{2}x-4$ is $x=10;\quad y=1$,	2 points	
thus the third vertex of the triangle is $C(10;\ 1)$.	1 point	
Total:	**7 points**	

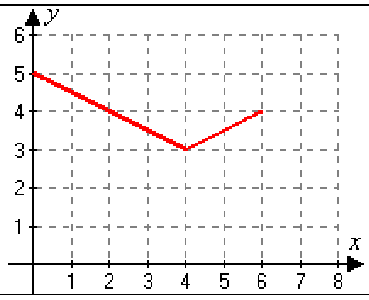
b)

Let T denote the foot of the altitude drawn from point C. From the right-angled triangle CTB, $tan\beta=0.1$.	3 points	*3 points should be awarded for determining any trigonometric function of β (e.g. from slope, cosine rule, etc.)*
Therefore $\beta\approx 5.71°$.	1 point	*If the trigonometric function of β is* **theoretically** *wrong no point is due for the angle.*
Total:	**4 points**	

2.		
a)		
A: true; B: false; C: true; D: true	4 points	*1 point should be given for every correct answer.*
Total:	**4 points**	
b)		
Altogether $2^4 = 16$ different ways of filling in the table are possible.	1 point	
Only 1 of them is correct.	1 point	
Therefore the probability is $\frac{1}{16} = 0.0625$.	1 point	*1 point should be awarded for any form of the correct answer.*
Total:	**3 points**	
c)		
There is love that never ends.	3 points	
Total:	**3 points**	
d)		
E.g. How many straight lines are determined by 17 points of the plane if no three points are collinear?	3 points	*Award 1 or 2 points if the import of the problem appears in the question but it is formulated imprecisely.*
Total:	**3 points**	

A	B	C	D
true	false	true	true

3.		
If the second term of the arithmetic progression is a_2 and its difference is d, then $a_2 - d + a_2 + a_2 + d = 60$,	2 points	*Altogether 2 points should be awarded for expressing the first condition using two variables.*
and hence $a_2 = 20$.	1 point	Or $a_1 + d = 20$.
The first three terms of the geometric progression are $84 - d$; 20; $20 + d$,	1 point	*Altogether 3 points should be awarded for reaching a quadratic equation.*
therefore $(84 - d)(20 + d) = 400$, or $\frac{20}{84 - d} = \frac{20 + d}{20}$.	2 points	
Rearranging the equation $d^2 - 64d - 1280 = 0$.	2 points	*For rearranging the equation.*
Hence $d_1 = -16$ or $d_2 = 80$.	2 points	*For solving the equation.*
$d_1 = -16$ is not a solution because the arithmetic progression is increasing.	1 point	*This is the 1 point lost if the candidate does not reject this case and gets two solutions.*
For $d_2 = 80$ the first three terms of the arithmetic progression are –60, 20, 100; which do give a correct solution.	1 point	*1 point for correctly listing the three numbers.*
From this, the three numbers 4, 20, 100 can be calculated, and they are indeed the first three terms of a geometric progression.	1 point	*1 point for correctly listing the three numbers.*
Total:	**13 points**	

4.		
a)		
	4 points	*4 points should be awarded for either using transformations of functions or for any other correct method.* *For an incomplete or wrong graph proportionately fewer points should be awarded.*
Total:	**4 points**	

b)		
The range is $[3;\ 5]$.	2 points	*Full credit should be given for any other form of the range.*
Total:	**2 points**	

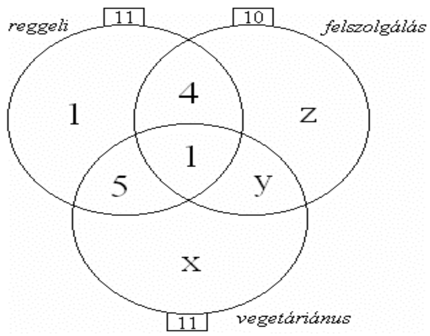
c)		
The solid of revolution is the frustrum of a cone.	3 points	*A good sketch is enough.*
The radii of the base circles are $R = 5$; $r = 3$. The height of the object is $h = 4$.	3 points	
The volume of the frustrum of a cone is $V = \frac{h\pi}{3}(r^2 + rR + R^2) = \frac{4\pi}{3}(25 + 15 + 9) =$ $= \frac{196\pi}{3} \approx 205.25$.	2 points	*The 2 points should also be awarded if no approximate values are calculated.*
Total:	**8 points**	

II.

Out of problems 5 to 9, do not assess the one indicated by the candidate.

5.

breakfast table service



vegetarian meals

a)

The Venn diagram above shows the numbers of the restaurants in the various categories.		*No sketch is required for the solution, full credit can also be achieved without the use of a diagram.*
Since only one restaurant provides all three services, therefore 1 should be written in the intersection of the three sets.	1 point*	
Since 5 restaurants provide breakfast as well as table service, 5 – 1 = 4 of them have breakfast and table service but no vegetarian meal.	1 point*	
Since 5 restaurants serve breakfast but no vegetarian meals, therefore 1 of them provides only breakfast.	1 point*	
Since 11 restaurants provide breakfast, therefore breakfast and vegetarian meals without table service are provided in 11 – 1 – 4 – 1 = 5 restaurants.	1 point*	
Since 11 restaurants provide vegetarian meals and 6 of them also provide breakfast, therefore 5 restaurants provide vegetarian meals but no breakfast.	1 point*	
Total:	**5 points**	

**These 1 points should be awarded for numbers written in the diagram, even without reasons given.*

b)

From the number of "vegetarian places" $y = 5 - x$, from that of places with table service: $z = x$.	2 points	
Hence the total number of restaurants is $11 + 2x + 5 - x = 18$,	1 point	

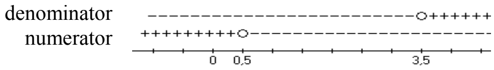
from which $x = 2$,	1 point	
therefore $y = 3$ (and $z = 2$).	1 point	The value of z is not needed.
Thus $y + 1 = 4$ restaurants serve vegetarian meals.	1 point	
Total:	**6 points**	

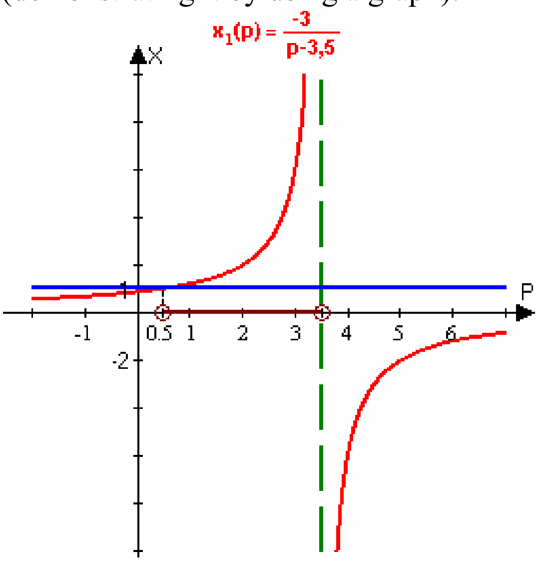
c)		
There are 18 restaurants altogether, 11 of which serve breakfast. Picking from box *A,* containing all the names, the chance of winning is $\frac{11}{18} \approx 0.61$	2 points	*2 points should be awarded for any correct form.*
Out of the 8 self-service restaurants 6 provide breakfast, therefore the chance of winning by picking from box *B* is $\frac{6}{8} = 0.75$,	2 points	*2 points should be awarded for any correct form.*
therefore it is better to pick from box *B*.	1 point	
Total:	**5 points**	

6.

a)		
Substituting the value $x = -2$: $f(-2) = (p - 3.5) \cdot 4 - 4(p - 2) + 6 =$ $= 4p - 14 - 4p + 8 + 6 = 0.$	2 points	*These 2 points should also be given if the candidate starts the solution with part b, assumes that $p \neq 3.5$, solves the equation, finds –2 as one of the roots and shows that it is also a root for $p = 3.5$.*
Total:	**2 points**	

b)		
For $p = 3.5$ the equation is not quadratic, there cannot be two roots, therefore $p \neq 3.5$.	1 point	
The roots of the equation are $x_{1,2} = \frac{-2(p-2) \pm \sqrt{4(p-2)^2 - 24(p-3.5)}}{2(p-3.5)} =$	1 point	
$= \frac{-p + 2 \pm \sqrt{p^2 - 10p + 25}}{p - 3.5} =$	1 point	
$= \frac{-p + 2 \pm (p - 5)}{p - 3.5} \Rightarrow$	2 points	
$x_1 = \frac{-3}{p - 3.5}$ and $x_2 = -2$	1 point	*Altogether 5 points for the roots of the parametric quadratic equation.*
The inequality $\frac{-3}{p - 3.5} > 1$ is to be solved.		

Rearranging the inequality $\frac{-p+0.5}{p-3.5}>0$.	2 points	
	2 points 2 points	
The inequality is satisfied for $0.5<p<3.5$.	2 points	*Altogether 8 points should be awarded for the solution of the inequality.*
Total:	**14 points**	
2 points should be awarded if the candidate only shows that, with $p \neq 3.5$, the sufficient condition for the existence of the two distinct roots is $p \neq 5$.		

Note: The graphical solution of the last section is:		
Using the monotonic nature of the function $x_1(p)$ (demonstrating it by using a graph): 	6 points	*4 points for the graph of $x_1(p)$.* *2 points for calculating the point intersection. (Also 2 points if the candidate reads the abscissa of the intersection and then checks it correctly. If the candidate reads it incorrectly or does not check it only 1 point should be given.)*
Equality holds for $0.5<p<3.5$.	2 points	*2 points for stating the solution.*

7.		
We have the square roots of perfect squares: $\sqrt{(\sin x-2)^2}+\sqrt{(\sin x+2)^2}=\sqrt{(\sin x+3.5)^2}$.	2 points	*For recognising the perfect squares.*
Taking the square roots: $\lvert\sin x-2\rvert+\lvert\sin x+2\rvert=\lvert\sin x+3.5\rvert$.	2 points	*A maximum of 4 points should be awarded altogether if the candidate omits the absolute values in taking square roots.*
Since $-1 \le \sin x \le 1$, therefore: $\left.\begin{array}{l}\sin x+2>0\\ \sin x-2<0\\ \sin x+3.5>0\end{array}\right\}$ *for* $\forall x \in \mathbf{R}$.	3 points	*3 points for examining the ranges of the functions. Altogether 5 points for the correct treatment of the absolute values.*
Therefore by eliminating the absolute value bars: $-\sin x+2+\sin x+2=\sin x+3.5$.	2 points	

$\sin x = \frac{1}{2}$.	1 point	
Hence $x_1 = \frac{\pi}{6} + 2k\pi$,	2 points	
or $x_2 = \frac{5\pi}{6} + 2k\pi$,	2 points	
where $k \in \mathbf{Z}$.	1 point	*
Checking: both sets of roots satisfy the original equation.	1 point	
Total:	**16 points**	

** Note :*

$x_1 = 30° + k \cdot 360°$ (1 point); $x_2 = 150° + k \cdot 360°$ (1 point); $k \in \mathbf{Z}$ (1 point)

or

$x_1 = 30°$; $x_2 = 150°$ (1 point)

or

$x_1 = 30° + k \cdot 2\pi$; $x_2 = 150° + k \cdot 2\pi$ (1 point); $k \in \mathbf{Z}$ (1 point)

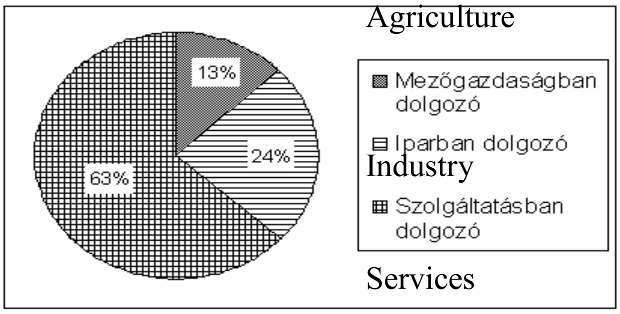
8.

a)

The total the labour force is $8500 \cdot 1.003 \approx 8526$ (thousand people).	2 points	
The unemployment rate is the same, therefore the number of the unemployed is $8526 \cdot \frac{595}{8500} \approx 597$ (thousand people).	2 points	
The number of people employed in the service sector is $5015 \cdot 1.02 = 5115$ (thousand people).	2 points	
The number of people employed in agriculture is $8526 - 597 - 1926 - 5115 = 888$ (thousand people).	1 point	
(table below)		*Maximum 5 points can be awarded for not rounding to the nearest thousand. 1 point should be subtracted per rounding error.*
Total:	**7 points**	

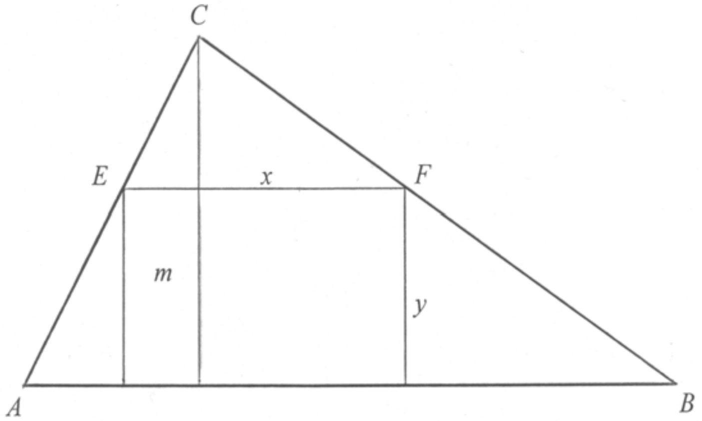
	2003. (*thousand people*)	2004. (*thousand people*)
Agriculture	1020	888
Industry	1870	1926
Services	5015	5115
Unemployed	595	597
Altogether	8500	8526

b)		
The total number of people employed in 2003-is 7905 thousand people.	1 point	*No point for the statement of 7905 on its own.*
The central angle of the sector representing people working in agriculture according to their proportion is $\frac{1020}{7905}\cdot 360° \approx 46°$.	1 point	*1 point should be given for stating each central angle, details of the calculation itself are not required.*
The central angle of the sector representing people working in industry is $\frac{1870}{7905}\cdot 360° \approx 85°$.	1 point	
(The central angle of the sector representing people working in the service sector is $\frac{5015}{7905}\cdot 360° \approx 228°$.)		

The distribution of the people employed in the various sectors in 2003. is as follows: 	2 points	*Writing the values of percentages into the pie chart is not required but the sectors should be identifiable.*
Total:	**5 points**	

c)		
$\frac{888}{1020} \approx 0.87$.	2 points	
The decrease is approximately 13 percent.	2 points	
Total:	**4 points**	

9.



To calculate the altitude drawn to side *AB* we can express the area of the triangle two different ways: $T=\sqrt{54\cdot 12\cdot 14\cdot 28}=504$.	2 points	
$T=\dfrac{42\cdot m}{2}$.	1 point	
From the equality of the different expressions of area: $m = 24$.	2 points	
Let x denote the side of the rectangle lying on side *AB*, and let y denote its other side. Triangles *ABC* and *EFC* are similar to each other since their sides are parallel.	2 points	
Because of the similarity: $\dfrac{x}{24-y}=\dfrac{42}{24}$,	2 points	
from which we get $y=\dfrac{168-4x}{7}$.	1 point	
The area of the triangle expressed as a function of x, where $x\in\,]0;\,42[$: $t(x)=xy=\dfrac{168x-4x^2}{7}$.	1 point	*1 point should be awarded even if the domain is not stated.*
It is sufficient to find the maximum of the function $\dfrac{7}{4}\cdot t(x)=42x-x^2$.	1 point*	*4 points should be awarded for any correct way of finding the maximum.*
By completing the square, the function can be written as $x\mapsto -(x-21)^2+441$.	1 point*	
The function has its maximum where the square is zero, that is $x = 21$.	1 point*	
$21\in\,]0;42[$, therefore it is a maximum point, indeed.	1 point*	
The other side of the rectangle is $y = 12$.	1 point	
Total:	**16 points**	

* *Note:* finding the maximum by differentiation: $t'(x)=\frac{168}{7}-\frac{8}{7}x$	1 point	
The derivative is zero for $x = 21$.	1 point	
$t''(x)=-\frac{8}{7}<0$, therefore $x = 21$ is a local maximum point.	1 point	
$21 \in \left]0;42\right[$, therefore the maximum is indeed at x = 21.	1 point	